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PROPERTIES OF M-PROJECTIVE RECURRENT RIEMANNIAN MANIFOLD

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Abstract: **- In this paper, analysis of some properties of the m-projective recurrent curvature tensor in Riemannian manifold has been done.**

Keywords: - Riemannian manifold, recurrent parameter, m-projective curvature tensor, Einstein manifold.

I. INTRODUCTION

An n-dimensional Riemannian manifold with Riemannian metric g be denoted by M.The Riemannian curvature and Riemannian connection is denoted by K and D respectively.A Riemannian manifold is recurrent if

(1.1) $(D_U K)X, Y, Z) = \alpha(U)\, K (X, Y, Z)$

A non zero 1-form α in (1.1) is known as recurrence parameter . The manifold reduces to symmetric manifold if 1-form α is zero in (1.1)

On contracting with respect to X equation (1.1), we get

(1.2) $(D_U Ric)(Y, Z) = \alpha(U)\, Ric(Y, Z).$

Thus from (1.2),we have

(1.3) $(D_U Q)(Y) = \alpha(U)Q(Y)$

Ricci operator of type (1,1) is denoted by Q, defined as

(1.4) $Ric(Y, Z) = g(Q(Y), Z).$

On contracting with respect to Y equation (1.3), we get

(1.5) $Ur = \alpha(U)r$

The scalar curvature is denoted by r.

II. M-PROJECTIVE CURVATURE TENSOR

G.P. Pokhariyal and R.S. Mishra [5] in 1971 defined a tensor field W^* on a Riemannian manifold as

(2.1) $$W^* (X, Y)Z = R(X, Y)Z - \frac{1}{2(n-1)} [Ric(Y,Z)X - Ric(X,Z)Y + g(Y, Z)QX - g(X, Z)QY].$$

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In Sasakian manifolds R.H.Ojha [3] defined and studied the properties of the m-projective curvature. It bridges the gap between the conformal curvature tensor,Coharmonic curvature tensor and Concircular curvature tensors was shown by him.

In this paper,we have considered a non flat n-dimensional smooth Riemannian manifold in which the M-projective curvature tensor W^* satisfies the following condition

(2.2) $$(D_u W^*)(X,Y)Z = \alpha(U)W^*(X,Y)Z$$

Manifold reduces to the m-projectively symmetric manifold if the 1-form α is zero.
P being the Projective curvature tensor is given by

(2.3) $$P(X, Y)Z = R(X, Y)Z - (1/n - 1)[Ric(Y, Z)X - Ric(X,Z)Y].$$

A projectively recurrent manifold a Riemannian manifold which obeys

(2.4) $$(D_U P)(X, Y)Z = \alpha(U)P(X, Y)Z$$

An Einstein manifold is a manifold which obeys

(2.5) $$Ric(X, Y) = kg(X, Y)$$

where k stands for a constant. From (2.5),we get

(2.6) $$Q(X) = kX.$$

On contracting the above equation with respect to X,we get

(2.7) $$r = nk.$$

III. M-PROJECTIVELY RECURRENT MANIFOLD

Theorem 3.1. *The constant curvature tensor r in an n-dimensional m-projectively recurrent Riemannian manifold, is given by*

(3.1) $$(n - 3)(Ur) + 2(2 - n)\alpha(U)r - 2n\alpha(QU) = 0.$$

Proof. Let Mn be an n-dimensional M-projectively recurrent manifold. Then from equations (2.1) and (2.2), it follows that

(3.2) $$(D_U R)(X, Y)Z = \alpha(U)R(X, Y)Z + \frac{1}{2(n-1)}[(D_U Ric(Y,Z)X - (D_U Ric)(X,Z)Y + g(Y,Z)(D_U Q)X - g(X,Z)(D_U Q)Y - \alpha(U)\{Ric(Y,Z)X - Ric(X,Z)Y + g(Y, Z)QX - g(Y, Z)QY \}].$$

Permuting equation (3.1) twice with respect to U,X, Y and adding the three equations and using Bianchi's second identity,we have

(3.3) $$\alpha(U)R(X, Y)Z + \alpha(X)R(Y,U)Z + \alpha(Y)R(U,X)Z + \frac{1}{2(n-1)}[(D_U Ric)(Y,Z)X - (D_U Ric)(X,Z)Y + g(Y,Z)(D_U Q)(X) - g(X,Z)(D_U Q)(Y) + (D_X Ric)(U,Z)Y - (D_X Ric)(Y,Z)U + g(U, Z)(D_X Q)(Y) - g(Y,Z)(D_X Q)(U) + (D_Y Ric)(X,Z)U - (D_Y Ric)(U,Z)X + g(X,Z)(D_Y Q)(U) - g(U, Z)(D_Y Q)(X) - \alpha(U)\{Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY \} - \alpha(X)\{Ric(U,Z)Y - Ric(Y,Z)U + g(U, Z)QY - g(Y,Z)QU\} - \alpha(Y)\{Ric(X,Z)U - Ric(U,Z)X + g(X,Z)QU - g(U, Z)QX\}] = 0.$$

On contracting the above equation with respect to X, we obtain

(3.4) $\alpha(U)Ric(Y,Z) - \alpha(Y)Ric(U,Z) + R'(Y, U,Z, \rho) + \frac{1}{2(n-1)}[(n-1)(DURic(Y,Z) + g(Y,Z)(Ur)$
$- g((DUQ)Y,Z) + (DY\,Ric)(U,Z) - (DURic)(Y,Z + \frac{1}{2}g(U, Z)(Y r) - \frac{1}{2}g(Y,Z)(Ur)$
$+ (1-n)(DY\quad Ric)(U,Z) + g((DYQ)U,Z) - g(U, Z)(Y r) + (1-n)\alpha(U)Ric(Y,Z)$
$+ \alpha(U)g(Y,Z)r + \alpha(U)g(Q(Y), Z) - \alpha(Y)Ric(U,Z) + \alpha(U)Ric(Y,Z)$
$- \alpha(Q(Y)g(U, Z) + \quad \alpha(Q(U)g(Y,Z) - \alpha(Y)g(R(U), Z)$
$+ (n-1)\alpha(Y)Ric(U,Z) + \alpha(Y)g(U, Z)r]=0$

Where vector field ρ is defined as

(3.5) $g(X, \rho) = \alpha(X)$

Factoring off Z in above,we have

(3.6) $\alpha(U)QY - \alpha(Y)QU - R(Y, U, \rho) + \frac{1}{2(n-1)}[(DYQ)U + (n-1)(DUQ)Y + Y(Ur) - (DUQ)Y$
$+ \frac{1}{2}U(Y r) - (DUQ)Y - \frac{1}{2}Y(Ur) + (1-n)(DYQ)U + (DY\,Q)U - U(Y r) - (n-1)\alpha(U)QY$
$- \alpha(U)(Y r) + \alpha(U)QY - \alpha(Y)QU + \alpha(U)QU - \alpha(Q(Y))U + \alpha(Y)Ur + \alpha(Q(U))Y$
$+ (n-1)\alpha(Y)QU] = 0.$

Or

$$R(Y, U, \rho) = \frac{1}{2(n-1)}[(n-3)(DUQ)Y + \alpha(U)QY - (n-3)(DYQ)U - \alpha(Y)QU + \alpha(Q(U))Y$$
$$- \alpha((3.6)\ Q(Y))U - \alpha(U)Y r + \alpha(Y)Ur].$$

On contracting (3.6) with respect to Y, we get

(3.7) $Ric(Y, \rho) = \frac{1}{2(n-1)}[(n-3)(Ur) - \frac{n-3}{2}(Ur) + (2-n)\alpha(U)r + (n-2)\alpha(QU)].$

Using(1.4) and (3.4) in equation (3.7),we get

$$(n-3)(Ur) + 2(2-n)\alpha(U)r - 2n\alpha(QU) = 0.$$

This completes the proof of the theorem.

Theorem 3.2. *The necessary and sufficient condition for an n-dimensional Ricci recurrent Riemannian manifold to be a recurrent manifold is that it is m-projectively recurrent manifold for the same recurrence parameter.*

Proof. Taking the covariant derivative of (2.1) with respect to *U*, we get

(3.8) $(D_U W^*)(X, Y)Z = (D_U R)(X, Y)Z - \frac{1}{2(n-1)}[(D_U Ric)(Y,Z)X$
$- (D_U Ric)(X,Z)Y + g(Y,Z)(D_U Q)(X) - g(X, Z)(D_U Q)(Y)].$

Let M^n be a Ricci recurrent Riemannian manifold,then from (1.2),(1.3) and (3.8),we have

(3.9) $(D_U W^*)(X, Y)Z = (D_U R)(X, Y)Z - \alpha(U)2(n-1)[Ric(Y,Z)X$
$- (Ric(X,Z)Y + g(Y,Z)QX - g(X, Z)QY].$
From(3.5) it is evident that if any one of the equations (1.1) and (2.5) hold then the second also hold.

Theorem 3.3. *In an Einstein manifold M^n the m-projective curvature tensor satisfies the following identity*

$(D_U W^*)(X, Y)Z + (D_X W(3.10) *)Y, U)Z + (DY□^*)(U,X)Z = 0.$

Proof. Using(2.7) and (2.8) in (2.1),it folows that

(3.11) $W^{*}(X, Y)Z = R(X, Y)Z - [k/(n-1)]\,[g(Y, Z)X - g(X, Z)Y]$.

Taking covariant derivative of the above with respect to U ,we get

(3.12) $(D_U W^{*})(X, Y)Z = (D_U R)(X, Y)Z.$

permuting equation(3.10) twice withrespect to U,X, Y ;adding the three equations
and using Bianchi's second identity,we get the required result.

Theorem 3.4. *Let* M^n *be an n-dimensional Ricci recurrent Riemannian manifold.Then* M^n *is m-projective recurrent if and only if it is a projectively recurrent manifold of the same recurence parameter.*

Proof. We have the following relation in projective curvature tensor and m-Projective curvature tensor

(3.13) $W^{*}(X, Y)Z = P(X, Y)Z + \frac{1}{2(n-1)}\,[Ric(Y,Z)X - Ric(X,Z)Y - g(Y, Z)QX + g(X, Z)QY]$.

Taking the covariant derivative of (3.10) with respect to U ,we get

(3.14) $$(D_U W^{*})(X, Y)Z = (DUP)(X, Y)Z + \frac{1}{2(n-1)}\,[(DURic)(Y,Z)X - (DURic)(X,Z)Y - g(Y,Z)(DUQX) + g(X, Z)(DUQY)].$$

Let M^n be a Ricci-recurrent Riemannian manifold,then from(1.2),(1.3) and

(3.12) it follows that

(3.15) $$(D_U W^{*}(X, Y)Z = (D_U P)(X, Y)Z + [\alpha(U)/2(n-1)][Ric(Y,Z)X - Ric(X,Z)Y - g(Y,Z)QX + g(X, Z)QY].$$

With the help of equations (3.12) and (3.13) it follows that if any one of the equations (2.6) and (2.2) hold then the second equation also holds.

IV. CONCLUSIONS

The constant curvature tensor r in an n-dimensional m-projectively recurrent Riemannian manifold, is given by $(n-3)(Ur) + 2(2-n)\alpha(U)r - 2n\alpha(QU) = 0$. The necessary and sufficient condition for an n-dimensional Ricci recurrent Riemannian manifold to be a recurrent manifold is that it is m-projectively recurrent manifold for the same recurrence parameter. M^n being an n-dimensional Ricci recurrent Riemannian manifold and is m-projective recurrent iff it is a projectively recurrent manifold of the same recurence parameter.

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