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CERTAIN CURVATURE CONDITIONS SATISFIED BY N(k)-QUASI EINSTEIN MANIFOLDS

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Abstract: **In present paper N(k)-quasi Einstein manifolds is studied and its existence is Proved by two non-trivial examples. A physical example of an N(k)-quasi-Einstein manifold is given. We study an N(k)-quasi-Einstein manifold satisfying certain curvature conditions like Ž(ξ, X).S = 0, and P (ξ, X) . C = 0. Also studied Ricci-pseudosymmetric N(k)-quasi-Einstein manifolds.**

.***Keywords:*** **Quasi Einstein manifold,N(k)-quasi Einstein manifold,projective curvature tensor, concircular curvature tensor, conformal curvature tensor,Ricci-pseudosymmetric manifold.**

I. INTRODUCTION

An Einstein manifold is a Riemannian or a semi-Riemannian manifold (Mn,g), n = dim M$\geq\in$, if the following condition holds on M, where S is the Ricci tensor and r denotes the scalar curvature of (Mn, g)

(1.1) $$S= \frac{r}{n} g$$

(1.1) is called the Einstein metric condition as mentioned in [4]. In Riemannian geometry as well as in general theory of relativity Einstein manifolds play an important role. By a curvature condition imposed on their Ricci tensor [4], Einstein manifolds form a natural sub-class of various classes of Riemannian or semi-Riemannian manifolds. The class of Riemannian manifolds (Mn, g) realizing the following relation consists of every Einstein manifold.

(1.2) $$S(X, Y) = ag(X, Y) + b \eta (X) \eta (Y)$$

where a, b are smooth functions and η is a non-zero 1-form such that

(1.3) $$g(X, \xi) = \eta (X), \qquad g(\xi, \xi) = \eta (\xi) = 1$$

Where X stands for all vector fields.

A quasi Einstein manifold [7] is a non-flat Riemannian manifold (Mn, g) (n > 2) if its Ricci tensor S of type (0,2) is not identically zero and the condition in (1.2) is satisfied. The unit vector field ξ is called the generator of the manifold and we call η the associated 1-form.

The study of exact solutions of the Einstein field equations and also during considerations of quasi-umbilical hypersur-faces of semi-Euclidean spaces, Quasi Einstein manifolds arose. In [15], Naschie on the theory of elementary particles and using Einstein's unified field equation, showed the expectation number of elementary particles of the standard model. In [14] Naschie also discussed possible connections between G¨odel's classical solution of Einstein's field equations and E-infinity. In the general theory of relativity quasi Einstein manifolds have some importance. The Robertson-Walker spacetime are quasi Einstein manifolds by [13].In general relativity [9] ,quasi Einstein manifold can be taken as a model of the perfect fluid spacetime.
Chaki [5], Guha [16], De and Ghosh [10, 11] and many others have studied quasi Einstein manifolds.In [20] Ozg¨ur studied

super quasi-Einstein manifolds and generalized quasi-Einstein manifolds [18]. In [17] Nagaraja studied N(k)-mixed quasi-Einstein manifolds.

The Riemannian curvature tensor of a Riemannian manifold M be denoted by R.
In [23] the k-nullity distribution N(k) of a Riemannian manifold M is defined by

N(k) . p $\rightarrow$Np(k) ={ $Z \in$ TpM : R(X,Y)Z = k[g(Y,Z)X - g(X, Z)Y] },
Where k is some smooth function. If the generator ξ belongs to some k-nullity distribution N(k), in a quasi Einstein manifold M, then M is said to be a N(k)-quasi Einstein manifold by [24]. By the following k is not arbitrary:

Lemma 2.1. [22] it follows that in an n-dimensional N(k)-quasi Einstein manifold

(1.4) k = [(a + b)/(n-1)]

Now, it is immediate to note that in an n-dimensional N(k)-quasi-Einstein manifold [22]

(1.5) R(X, Y) ξ = [(a + b)/(n-1] [η (Y)X - η(X)Y].

which is equivalent to

R(X, ξ)Y = [(a + b)/(n-1)] [η (Y)X - g(X, Y) ξ] = -R(ξ,X)Y.

From (1.5) we get

R(ξ,X) ξ = [(a + b)/(n-1)] [η (X) ξ - X].

In [24], it was shown that an n-dimensional conformally flat quasi Einstein manifoldis an N [(a + b)/(n-1)] quasi Einstein manifold and in particular a 3-dimensional quasiEinstein manifold is an N [(a + b)/(2)] quasi Einstein manifold. In differential geometry and also in general theory of relativity the conformal curvature tensor plays an important role. Defination of the Weyl conformal curvature tensor C of a Riemannian manifold (Mn,g) (n > 3) is :

C(X, Y)Z = R(X, Y)Z [n-1)/2] [g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X-S(X,Z)Y] + [r/(n-1)(n-2)]
[g(Y,Z)X - g(X,Z)Y]

where Q is the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor S that is, g(QX, Y) =S(X, Y) and r is the scalar curvature. For the dimension n = 3, then the conformal curvature tensor vanishes identically.The projective curvature tensor P and the concircular curvature tensor Ž in a Riemannian manifold (Mn, g) are defined by [25]

P(X, Y)W = R(X, Y)W-[1/(n-1)] [S(Y,W)X - S(X,W)Y],

Ž (X, Y)W = R(X, Y)W - [r/(n-1)] [g(Y,W)X - g(X,W)Y], respectively.

It is proved in [24] by the authors that conformally flat quasi-Einstein manifolds are certain N(k)-quasi-Einstein manifolds. In [23] the derivation conditions R(ξ,X) . R = 0 and R(ξ,X) . S = 0 have been studied, where R denotes the curvature tensor and and S denotes Ricci tensor. Further in [22] ¨Ozg¨ur and Tripathi studied the derivation conditions Ž (ξ,X).R = 0 and Ž (ξ,X) . Ž = 0 on N(k)-quasi-Einstein manifold where Ž is the concircular curvature tensor .It was) in [22] proved that k =[(a+b)/(n-1, for an N(k)-quasi-Einstein manifold. The condition R. P = 0, P . S = 0 and P .P = 0 for an N(k)-quasi-Einstein manifolds, where P denotes the projectivecurvature tensor ,studied by ¨Ozg¨ur [19] and some physical examples of N(k)-quasi-Einstein manifolds are given. ¨Ozg¨ur and Sular [21] in 2008,studied N(k)-quasi-Einstein manifold satisfying R.C = 0 and R. Č = 0, where C represents the conformal curvature tensor and Č represents the quasi-conformal curvature tensor. In continuation of previous studies this paper is presented.In this paper after preliminaries some examples of N(k)-quasi-Einstein manifolds are discussed. In this paper some we give a physical example of an N(k)-quasi-Einstein manifold and we study N(k)-quasi-Einstein manifold satisfying Ž (ξ,X) . S = 0 in Section 5. Section 6 deals with N(k)-quasi-Einstein manifolds satisfying P(ξ,X) . C = 0. In and Section 7, we study N(k)-quasi-Einstein manifolds satisfying Ž (ξ,X) . C = 0. Finally, we study Ricci-pseudosymmetric N(k)-quasi-Einstein manifolds.

II. PRELIMINARIES

From (1.2) and (1.3) it follows that

r = an + b and QX = (a + b)X,

S(X, ξ) = k(n - 1) η (X),

Where r is the scalar curvature and Q is the Ricci operator.

In an n-dimensional N(k)-quasi-Einstein manifold M, the projective curvature tensor P, the concircular curvature tensor Ž and the conformal curvature tensor C satisfy the following relations:

P(X, Y) ξ = 0,

(2.1) η (P(ξ,X)Y = $[\frac{b}{n-1}]$ [g(X, Y) $\xi - \eta$ (X) η (Y) ξ],

(2.2) P(X, Y)Z) = $[\frac{b}{n-1}]$ [g(Y,Z) η (X)- g(X,Z) η (Y)],

(2.3) $Z(X,Y)Z = [k - \frac{r}{n(n-1)}][g(Y,Z)X - g(X,Z)Y]$,

(2.4) $Z(\xi,Y)Z = [k - \frac{r}{n(n-1)}][g(Y,Z)\,\xi - \eta(Z)Y]$,

(2.5) C(X, Y)Z = $k - \frac{r}{(n-1)(n-2)}$ [g(Y,Z)X - g(X,Z)Y] –[1/(n – 2)] [(2a + b)g(Y,Z)X - (2a + b)g(X,Z)Y + b η (Y) η (Z)X- b η (X) η (Z)Y],

(2.6) η (C(X, Y)Z) = 0,

(2.7) $C(\xi,Y)Z = -\frac{b}{n-2}[\eta(Y)\eta(Z)\,\xi - \eta(Z)Y]$,

for all vector fields X, Y , Z on M.

III. EXAMPLES OF *N*(*K*)-QUASI EINSTEIN MANIFOLDS

Example 3.1. An N(k)-quasi Einstein manifold consists of special para-Sasakian manifold with vanishing D-concircular curvature tensor V. A Riemannian manifold, Mn having a unit concircular vector field ξ such that $\nabla_x \xi = \varepsilon(-X + \eta(X)\xi),\ \eta(X) = g(X.\xi),\ \varepsilon = \pm 1$, then Mn is called a special para-Sasakian manifold [1–3]. The notion of a D-concircular curvature tensor V is recently introduced by Chuman [8]. V is given by the following equation

V (X, Y,Z,W) = Ř (X, Y,Z,W) + $\frac{r+2(n-1)}{(n-1)(n-2)}$ [g(X,Z)g(Y,W) - g(Y,Z)g(X,W)]

(3.1) $$-\frac{r+n(n-1)}{(n-1)(n-2)}[g(X,Z)\,\eta(Y)\,\eta(W) - g(Y,Z)\eta(X)\eta(W) + g(Y,W)\eta(Z)\eta(X) - g(X,W)\,\eta(Y)\,\eta(Z)],$$

Where Ř (X,Y,Z,W) = g(R(X, Y)Z,W)) for R(X, Y)Z the curvature tensor of type (1, 3). If V (X, Y,Z,W) = 0, then from (3.1) it follows that

(3.2) $$\check{R}(X,Y,Z,W) = \frac{r+2(n-1)}{(n-1)(n-2)}[g(Y,Z)\,g(X,W) - g(X\text{-}Z)\,g(Y\text{-}W)] + \frac{r+n(n-1)}{(n-1)(n-2)}[g(X,Z)\,\eta(Y)\,\eta(W)\,\eta\,g(Y,Z)\,\eta(X)\,\eta(W) + g(Y,W)\,\eta(Z)\,\eta(X) - g(X,W)\,\eta(Y)\,\eta(Z)],$$

Putting X = W = e_i in (3.2) where $\{e_i\}$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over i, $1 \leq i \leq n$, we get

$S(Y,Z) = a\,g(Y,Z) + b\,\eta(Y)\,\eta(Z)$,

where $a = \frac{r+n-1}{n-1}$ and $b = -\frac{r+n(n-1)}{n-1}$. Therefore, $\frac{a+b}{n-1} = -1$. Hence a special para-Sasakian manifold with vanishing D-concircular curvature tensor is an N(-1)-quasi-Einstein manifold.

IV. A PHYSICAL EXAMPLE OF AN *N*(*K*)-QUASI-EINSTEIN MANIFOLD

Example 4.1 An N(k)-quasi-Einstein manifold in general relativity by the coordinate free method of differential geometry is concerned with this example . In this method of study the space time of general relativity is regarded as a connected four-dimensional semi-Riemannian manifold (M4, g) with Lorentzian metric g with signature (-,+,+, +). With the study of causal character of vectors of the manifold begins the geometry of the Lorentzian manifold. The Lorentzian manifold becomes a convenient choice for the study of general relativity due to this causality.

Here we consider a perfect fluid $(PRS)_4$ spacetime of non-zero scalar curvature and having the basic vector field U as the time like vector field of the fluid, that is,g(U,U) = -1. An n-dimensional semi-Riemannian manifold is said to be pseudo Ricci-symmetric [6] if the Ricci tensor S satisfies the condition

$(\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + A(Y)S(X,Z) + A(Z)S(Y,X)$.

Such a manifold is denoted by $(PRS)_n$.

For the perfect fluid spacetime, we have the Einstein equation without cosmological constant as

(4.1) $$S(X,Y) - \frac{1}{2}rg(X,Y) = \kappa T(X,Y),$$

where κ is the gravitational constant, T is the energy-momentum tensor of type (0, 2) given by

(4.2) $T(X,Y) = (\sigma + p)B(X)B(Y) + pg(X,Y)$,

with σ and p as the energy density and isotropic pressure of the fluid respectively.

Using (4.2) in (4.1) we get

(4.3) $S(X, Y) - 12rg(X, Y) = \kappa[(\sigma + p)B(X)B(Y) + pg(X, Y)]$.

Taking a frame field and contracting (4.3) over X and Y we have

(4.4) $r = \kappa(\sigma - 3p)$.

Using (4.4) in (4.3) yields

(4.5) $S(X, Y) = \kappa[(\sigma + p)B(X)B(Y) + \frac{(\sigma - p)}{2} g(X, Y)]$.

Putting Y = U in (4.5) and since g(U,U) = -1, we get

(4.6) $S(X,U) = -\frac{\kappa}{2}[\sigma + 3p]B(X)$.

Again for $(PRS)_n$ spacetime [6], S(X,U) = 0. This condition will be satisfied by the equation (4.6) if

(4.7) $\sigma + 3p = 0$ as $\kappa \neq 0$ and $A(X) \neq 0$.

Using (4.4) and (4.7) in (4.5) we see that

$S(X,Y) = \frac{r}{3}[B(X) B(Y) + g(X, Y)]$.

Thus we can state the following:

An $N(\frac{2r}{9})$-quasi-Einstein manifold is a perfect fluid pseudo Ricci-symmetric spacetime.

V. N(K)-QUASI EINSTEIN MANIFOLD SATISFYING Ž (ξ,X) . S = 0

In this section we consider an n-dimensional N(k)-quasi-Einstein manifold M satisfying the condition (Ž (ξ,X) . S)(Y,Z) = 0.

Putting Z = ξ we get

(5.1) S(Ž (ξ,X)Y, ξ + S(Y, Ž(ξ,X) ξ) = 0.

Using (1.2), (2.3) and (2.4) in (5.1) we get

(5.2) S(Ž (ξ ,X)Y, ξ) = $[k - \frac{r}{n(n-1)}]$(a + b)[g(X,Y) - $\eta(X)\eta(Y)$],

and

(5.3) S(Y,Ž (ξ,X) ξ) = $[k - \frac{r}{n(n-1)}][($a + b) $\eta(X)\eta(Y)$- S(X, Y)].

Using (5.2) and (5.3) in (5.1), we obtain

$[k - \frac{r}{n(n-1)}][($a + b)g(X, Y) - S(X, Y)] = 0.

Therefore, either the scalar curvature of M is kn(n-1) or, S = (a+b)g which implies that M is an Einstein manifold. But this contradicts the definition of quasi-Einstein manifold. The converse is trivial.

Thus we can state the following:

Theorem 5.1. The condition Ž (ξ,X) . S = 0 is satisfied by an n-dimensional N(k)-quasi-Einstein manifold M if and only if the scalar curvature is kn(n - 1).

VI. N(K)-QUASI-EINSTEIN MANIFOLD SATISFYING P(ξ,X) . C = 0

In this section we consider an n-dimensional N(k)-quasi-Einstein manifold satisfying the condition

(P(ξ,X). C)(Y,Z)W = 0

Then we have

(6.1) P(ξ,X)C(Y,Z)W-C(P(ξ,X)Y,Z)W-C(Y, P(ξ,X)Z)W-C(Y,Z)P(ξ,X)W = 0.

Using (2.1) in (8.1) we obtain

$\frac{b}{n-1}$ [g(X,C(Y,Z)W) ξ - η (X) η (C(Y,Z)W) ξ −- C(g(X, Y) ξ - η (X) η (Y) ξ,Z)W
- C(Y, g(X, Y) ξ - η(X) η (Z) ξ)W- C(Y,Z)(g(X,W) ξ - η(X) η (W) ξ)] = 0,

which implies either b = 0, or

(6.2) g(X,C(Y,Z)W) ξ - η (X) η (C(Y,Z)W) ξ - g(X, Y)C(ξ,Z)W
+ η (X) η (Y)C(ξ,Z)W- g(X,Z)C(Y, ξ)W + η (X) η (Z)C(Y, ξ)W
- g(X,W)C(Y,Z) ξ + η (X) η (W)C(Y,Z) ξ = 0,

holds on M. Since b $\neq$ 0, hence (8.2) holds.

Taking inner product with ξ and using (2.5)–(2.7) in (8.2) we obtain
g(X,C(Y,Z)W) = 0.

Thus we can state the following:

Theorem 6.1. An n-dimensional N(k)-quasi-Einstein manifold M satisfiesthe condition P(ξ,X) . C = 0 if and only if the manifold is conformally flat.

VII. N(K)-QUASI-EINSTEIN MANIFOLD SATISFYING Ž (ξ,X) . C = 0

In this section we consider an n-dimensional N(k)-quasi-Einstein manifold satisfying the condition

(Ž(ξ,X) . C)(Y,Z)W = 0.

Then we have

(7.1) Ž(ξ,X)C(Y,Z)W -C(Ž (ξ,X)Y,Z)W -C(Y, Ž (ξ,X)Z)W -C(Y, Ž)(ξ,X)W = 0.

Using (2.4) in (9.1) we obtain µ

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(7.2) $k - rn(n-1)\,[g(X,C(Y,Z)W)\xi - \eta(C(Y,Z)W)X - g(X,Y)C(\xi,Z)W$
$+ \eta(Y)C(X,Z)W - g(X,Z)C(Y,\xi)W + \eta(Z)C(Y,X)W$
$- g(X,W)C(Y,Z)\xi + \eta(W)C(Y,Z)X] = 0,$

which gives either $r = kn(n-1)$ or

$$g(X,C(Y,Z)W)\xi - \eta(C(Y,Z)W)X - g(X,Y)C(\xi,Z)W + \eta(Y)C(X,Z)W$$
$$-g(X,Z)C(Y,\xi)W + \eta(Z)C(Y,X)W - g(X,W)C(Y,Z)\xi + \eta(W)C(Y,Z)X] = 0.$$

Taking inner product with ξ and using (2.5)–(2.7) in (9.2) we obtain

$$g(X,C(Y,Z)W) = 0.$$

Thus we can state the following.

Theorem 7.1. The condition $\check{Z}(\xi,X)\,.\,C = 0$ is satisfied by an n-dimensional N(k)-quasi-Einstein manifold M if and only if the manifold is conformably flat.

VIII. RICCI-PSEUDOSYMMETRIC N(K)-QUASI-EINSTEIN MANIFOLDS

An n-dimensional Riemannian manifold (Mn, g) is called Ricci-pseudosymmetric[12] if the tensors R . S and Q(g, S) are linearly dependent, where

(8.1) $(R(X,Y)\,.\,S)(Z,W) = -S(R(X,Y)Z,W) - S(Z,R(X,Y)W),$

(8.2) $Q(g,S)(Z,W,X,Y) = -S((X \wedge Y)Z,W) - S(Z,(X \wedge Y)W),$

and

(8.3) $(X \wedge Y)Z = g(Y,Z)X - g(X,Z)Y,$ (10.3)

for vector fields X, Y,Z,W on Mn.

The condition of Ricci-pseudosymmetry is equivalent to

$$(R(X,Y)\,.\,S)(Z,W) = L_s Q(g,S)(Z,W,X,Y),$$

holding on the set

$$U_s = \{x \in M : S \neq \frac{r}{n} \text{ at } x\},$$

where Ls is some function on Us. If R.S = 0 then Mn is called Ricci-semisymmetric.
Every Ricci-semisymmetric manifold is Ricci-pseudosymmetric but the converse is not true [12].

Let us assume that the manifold under consideration is Ricci-pseudosymmetric.
Then with the help of (10.1)–(10.3) we can write

(8.4) $S(R(X,Y)Z,W) + S(Z,R(X,Y)W) = L_s\{g(Y,Z)S(X,W) - g(X,Z)S(Y,W)$
$+ g(Y,W)S(X,Z) - g(X,W)S(Y,Z)g\}.$

Using (1.2) and (1.4) in (10.4) we obtain

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(8.5) $\left[\frac{b(a+b)}{n-1} - bL_s\right]$ {g(Y,Z)η(X)η(W) - g(X,W)η(Y)η(Z)+ g(Y,W)η(X)η(Z) - g(X,Z)η(Y)η(W)g} = 0.

Putting Y = Z = ξ in (10.5) we have

(8.6) $\left[\frac{b(a+b)}{n-1} - bL_s\right]$ η(X)η(W) - g(X,W)] = 0.

Again putting X = W = e_i in (10.6), where $\{ e_i \}$, (i = 1, 2, … , n) is an orthonormal basis of the tangent space at any point of the manifold and then taking the sum for 1 $\leq$ i $\leq$ n, we obtain

·b(a + b)n - 1- bLs¸(1 - n) = 0,

which implies that Ls = $\frac{a+b}{n-1}$. Thus we can state the following:

Theorem 8.1.Iff a + b = 0 then a Ricci-pseudosymmetric N(k)-quasi-Einstein manifold is a Ricci-semisymmetric manifold.
curvature is kn(n - 1). condition Ž(ξ,X) . S = 0 if and only if the scalar curvature is kn(n - 1).the condition Ž(ξ,X) . S = 0 if and only if the scalar

IX. CONCLUSION

During the study of exact solutions of the Einstein field equations as well as during considerations of quasi-umbilical hyper surfaces of semi-Euclidean spaces arose Quasi-Einstein manifolds. In the introduction the importance of an N(k)-quasi-Einstein is presented. We prove that a special para Sasakian manifold with vanishing D-concircular curvature tensor V is an N(k)-quasi-Einstein manifold. A physical example of N(k)-quasi-Einstein manifolds is given. N(k)-quasi-Einstein manifolds satisfying the curvature conditions has been considered Ž(ξ,X) . S = 0, P(ξ,X) . Ž = 0, Ž(ξ,X) . P = 0, P(ξ,X) . C = 0 and Ž(ξ,X) . C = 0, where P, Ž, C, S are projective curvature tensor, concircular curvature tensor, conformal curvature tensor and Ricci tensor respectively. Finally we prove that a Ricci-pseudosymmetric N(k)- quasi-Einstein manifold is a Ricci-semi symmetric manifold under certain condition.

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REFERENCES

[1]. T. Adati, G. Chuman, Special para-Sasakian manifolds D-conformal to a manifold of constant curvature, TRU Math. 19 (1983), 179–193.

[2]. T. Adati, G. Chuman, Special para-Sasakian manifolds and concircular transformations,TRU Math. 20 (1984), 111–123.

[3]. T. Adati, T. Miyazaw, On para contact Riemannian manifolds, TRU Math. 13 (1977), 27–39.

[4]. A.L. Besse, Einstein Manifolds, Ergeb. Math. Grenzgeb., 3. Folge, Bd. 10, Springer-Verlag, Berlin, Heidelberg, New York, 1987.

[5]. M.C. Chaki, On generalized quasi-Einstein manifolds, Publ. Math. Debrecen 58 (2001),683–691.

[6]. M.C. Chaki, On pseudo Ricci-symmetric manifolds, Bulg. J. Physics 15 (1988), 526–531.

[7]. M.C. Chaki, R.K. Maity, On quasi Einstein manifolds, Publ. Math. Debrecen 57 (2000),297–306.

[8]. G. Chuman, D-conformal changes in para-Sasakian manifolds, Tensor, N.S. 39 (1982), 117–123.

[9]. U.C. De, G.C. Ghosh, On quasi Einstein and special quasi Einstein manifolds, Proc. Intern.Conf. Math. and its Applications, Kuwait University, April 5-7, 2004, 178–191.

[10]. U.C. De, G.C. Ghosh, On quasi-Einstein manifolds, Period. Math. Hungar. 48 (2004), 223–231.

[11]. U.C. De, G.C. Ghosh, On conformally flat special quasi-Einstein manifolds, Publ. Math.Debrecen 66 (2005), 129–136.

[12]. R. Deszcz, On pseudosymmetric spaces, Bull. Soc. Math. Belg. S´er. A 44 (1992), 1–34.

[13]. R. Deszcz, M. Hotlos, Z. Senturk, On curvature properties of quasi-Einstein hypersurfaces in semi-Euclidean spaces, Soochow J. Math. 27 (2001), 375–389.

[14]. M.S. El Naschie, G¨odel universe, dualities and high energy particle in E-infinity, Chaos, Solitons and Fractals 25 (2005), 759–764.

[15]. M.S. El Naschie, Is Einstein's general field equation more fundamental than quantum field theory and particle physics?, Chaos, Solitons and Fractals 30 (2006), 525–531.

[16]. S.R. Guha, On quasi-Einstein and generalized quasi-Einstein manifolds, Facta Univ. Niˇs 3 (2003), 821–842.

[17]. H.G. Nagaraja, On N(k)-mixed quasi-Einstein manifolds, Europ. J. Pure Appl. Math. 3(2010), 16–25.

[18]. C. ¨Ozg¨ur, On a class of generalized quasi-Einstein manifolds, Applied Sciences, Balkan Society of Geometers, Geometry Balkan Press, 8 (2006), 138–141.

[19]. C. ¨Ozg¨ur, N(k)-quasi Einstein manifolds satisfying certain conditions, Chaos, Solitons andFractals 38 (2008), 1373–1377.

[20]. C. ¨Ozg¨ur, On some classes of super quasi-Einstein manifolds, Chaos, Solitons and Fractals40 (2009), 1156–1161.

[21]. C. ¨Ozg¨ur, S. Sular, On N(k)-quasi-Einstein manifolds satisfying certain conditions, BalkanJ. Geom. Appl. 13 (2008), 74–79.

[22]. C. ¨Ozg¨ur, M.M. Tripathi, On the concircular curvature tensor of an N(k)-quasi Einstein manifold, Math. Pannon. 18 (2007), 95–100.

[23]. S. Tanno, Ricci curvatures of contact Riemannian manifolds, Tohoku Math. J. 40 (1988),441–448.

[24]. M.M. Teipathi, J.S. Kim, On N(k)-quasi Einstein manifolds, Commun. Korean Math. Soc.22 (2007), 411–417.

[25]. K. Yano, M. Kon, Structures on Manifolds, Series in Pure Mathematics, 3, World Scientific Publishing Co., Singapore, 1984.

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